$21/40 = 53\%$

MATH 124.3 (4) Winter 2002–2003: CALCULUS II

Test One – January 30, 2003 – 75 Minutes

Part I

No books, notes or calculators are allowed. This part is multiple-choice and computer-marked. Use a soft pencil. Encode your student number in the upper left corner of the opscan sheet. Print your name in the space indicated at the top of the opscan sheet. Return only the opscan sheet. Keep the question sheet and the scratch booklet. Each answer is worth 1 point. The possible answers for each question are

A) 0 *B*) 1 *C*) 2 *D*) 3 *E*) 4 *F*) 5 *G*) 6 *H*) 7 *I*) 8 *J*) 9

If $\dfrac{17-i}{1-3i} = a + bi$ (where $i^2 = -1$) then

1) $a = 2$ **2)** $b = 5$

Write the complex number $e^{\frac{3\ln 2}{2}+\frac{\pi}{4}i}$ in the form $c + di$:

3) $c = 4$ **4)** $d = 4$

Consider the sum $\sum_{k=0}^{3} 2^k$. If the first term is a, the last term is b, and the value of the sum is $10c + d$ then

5) $a = 1$ **6)** $b = 8$

7) $c = 1$ **8)** $d = 5$

If the area under the graph of $y = x^2$ between $x = 2$ and $x = 5$ is estimated using 3 equal subintervals and right endpoints then the result is $10a + b$ where

9) $a = 3$ **10)** $b = 9$

If the limit $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{2}{n}\sqrt{7+\frac{2k}{n}}$ represents the integral $\int_a^b \sqrt{x}\,dx$ using n equal subintervals and right endpoints then

11) $a = 4$ **12)** $b = 4$

Given that $\int_0^2 f(x)dx = 1$, $\int_0^2 g(x)dx = 3$, $\int_0^1 f(x)dx = 2$, and $\int_0^1 g(x)dx = 1$, find $I = \int_1^2 [f(x) + 2g(x)]\,dx$:

13) $I = 3$

MATH 124.3 (4) Winter 2002–2003: CALCULUS II

Test One – January 30, 2003 – 75 Minutes

Part II

No books, notes or calculators. This part is long-answer and hand-marked. Show your work.

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1. Evaluate the following limits: [2 points per part]

   $\ln 5 - \ln 3$ (a) $\lim_{t\to 0} \frac{5^t - 3^t}{t}$,

   0 (b) $\lim_{x\to+\infty} x^2 e^{-x}$,

   $-\infty$ (c) $\lim_{\theta\to\pi} \frac{\tan\theta}{\theta}$,

   ∞ (d) $\lim_{x\to 0}(1+ax)^{\frac{b}{x}}$ where a, b are real numbers,

   ? (e) $\lim_{x\to+\infty} x[\ln(x+3) - \ln x]$.

$\sqrt{3}/2 + i/2$ 2. Find all the cubic roots of $-i$ (where $i^2 = -1$). Sketch them on the complex plane. [2 points]

$0+i$ 3. Use De Moivre's Formula to find $(1-i)^{10}$. [2 points]

$26/3$ 4. Use the definition of the definite integral to evaluate $\int_1^3 x^2\,dx$ [you may use the formulas $\sum_{k=1}^n k = \frac{n(n+1)}{2}$ and $\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$]. Sketch the area represented by this integral. [5 points]

$2x\sqrt{1+x^6}$ 5. Let $y = \int_0^{x^2} \sqrt{1-t^3}\,dt$. Find $\frac{dy}{dx}$. [2 points]

✓ 6. Find the following integrals: [1 point per part]

   $\ln 2 - \ln 1$ (a) $\int_1^2 \frac{1}{x}\,dx$,

   $1/2$ (b) $\int_0^{\frac{\pi}{3}} \sin t\,dt$,

   ? (c) $\int \frac{x^2+1}{\sqrt{x}}\,dx$,

   1 (d) $\int_{-1}^1 |x|\,dx$.

13 7. The linear density of a rod of length 4 m is given by $\rho(x) = 9 + 2\sqrt{x}$ measured in kg/m, where x is measured in m from one end of the rod. Find the total mass of the rod. [2 points]

MATH 124.3 (4) Winter 2002–2003: CALCULUS II

Test Two – March 6, 2003 – 75 Minutes

Part I

No books, notes or calculators are allowed. This part is multiple-choice and computer-marked. Use a soft pencil. Encode your student number in the upper left corner of the opscan sheet. Print your name in the space indicated at the top of the opscan sheet. Return only the opscan sheet. Each answer is worth 1 point. The possible answers for each question are

$A)\,0 \quad B)\,1 \quad C)\,2 \quad D)\,3 \quad E)\,4 \quad F)\,5 \quad G)\,6 \quad H)\,7 \quad I)\,8 \quad J)\,9$

If the area between $y = x^2 + 2$, $y = 2x + 5$, $x = 0$, and $x = 6$ is given by $A = 10a + b$ then

1) $a =$ 3 **2)** $b =$ 6

A rectangular aquarium 2m long, 1.5m wide, and 1m deep is full of water. Find the work needed to pump half of the water out of the aquarium through an outlet at the top. Express your answer in kJ and round it to the nearest integer.

3) $W =$ 4

Suppose f is a continuous function on $[1, 3]$ and $\int_1^3 f(x)\,dx = 4$. Give a value y that is guaranteed to be taken on by f at some point in $[1, 3]$.

4) $y =$ 2

If $\displaystyle\int_1^2 \frac{\ln x}{x^2}\,dx = \frac{a}{b} - \frac{c}{d}\ln 2$ (in lowest terms) then

5) $a =$ 3 **6)** $b =$ 4

7) $c =$ 1 **8)** $d =$ 2

If $\displaystyle\int_1^2 \frac{dx}{x^3\sqrt{x^2-1}} = \frac{a}{b}\pi + \frac{c}{d}\sqrt{3}$ (in lowest terms) then

9) $a =$ 1 **10)** $b =$ 1

11) $c =$ 1 **12)** $d =$ 2

please turn over.....

MATH 124.3 (4) Winter 2002–2003: CALCULUS II

Test Two – March 6, 2003 – 75 Minutes

Part II

No books, notes or calculators. This part is long-answer and hand-marked. Show your work.

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1. The region between $y = x^4$ and $y = 1$ is revolved about the line $y = 2$. Sketch the region, the solid of revolution, and a typical "washer". Find the volume of the solid by the "washer" method. [5 points]

2. The region between $y = \cos x$, $y = 0$, $x = 0$, and $x = \frac{\pi}{2}$ is revolved about the y-axis. Sketch the region, the solid of revolution, and a typical "cylindrical shell". Find the volume of the solid by the "cylindrical shell" method. [5 points]

3. Find the average value of the function $f(x) = \sin^2 x$ on the interval $[0, 2\pi]$. [2 points]

4. Find the following integrals:

   (a) $\int e^{2\theta} \sin 3\theta \, d\theta$, [4 points]

   (b) $\int \sin^3 t \sqrt{\cos t} \, dt$, [3 points]

   (c) $\int \sec^4 x \, dx$, [4 points]

   (d) $\int \sqrt{1 - 4x^2} \, dx$. [5 points]

MATH 124.3 (4) Winter 2002–2003: CALCULUS II

Test Three – April 3, 2003 – 75 Minutes

Part I

No books, notes or calculators are allowed. This part is multiple-choice and computer-marked. Use a soft pencil. Encode your student number in the upper left corner of the opscan sheet. Print your name in the space indicated at the top of the opscan sheet. Each answer is worth 2 points.

1) Which of the following integrals is **not** improper?

A) $\int_0^1 \frac{1}{x^2}\,dx$ B) $\int_0^\infty \sin x\,dx$ C) $\int_{-2}^1 \frac{1}{x^2-1}\,dx$

D) $\int_{-\infty}^\infty \frac{1}{x^2+1}\,dx$ E) $\int_0^2 \frac{1}{\sqrt{x+1}}\,dx$

2) Evaluate $\int_0^2 \ln x\,dx$.

A) 0 B) $2\ln 2$ C) $2\ln 2+2$ D) $2\ln 2-2$ E) divergent

3) Evaluate $\int_{-2}^3 \frac{1}{x^2}\,dx$.

A) 0 B) $\frac{1}{6}$ C) $\frac{5}{6}$ D) $-\frac{5}{6}$ E) divergent

4) Which of the following functions is the solution of the initial-value problem

$$x^2y' + xy = 1, \quad y(1) = 2\ ?$$

A) $y = \frac{\ln x}{x}$ B) $y = \frac{2}{x}$ C) $y = \frac{\ln x}{x} + \frac{1}{x}$

D) $y = \frac{\ln x}{x} + \frac{2}{x}$ E) $y = \frac{\ln x}{x} + \frac{C}{x}$

For the rest of the questions in Part I, the possible answers are

A) 0 *B)* 1 *C)* 2 *D)* 3 *E)* 4 *F)* 5 *G)* 6 *H)* 7 *I)* 8 *J)* 9

If the length of the curve given by $y = \frac{1}{3}(x^2+2)^{3/2}$, $0 \le x \le 1$, is $\dfrac{a}{b}$ (in lowest terms), then

5) $a =$ 4 **6)** $b =$ 7

If the arc of the curve $2y = x^2$ between the points $(0,0)$ and $(2,2)$ is revolved about the y-axis, then the area of the resulting surface of revolution is given by $\dfrac{a}{b}\pi(c\sqrt{c}-1)$ (in lowest terms), where

7) $a =$ **8)** $b =$ **9)** $c =$

..continued..

MATH 124.3 (4) Winter 2002–2003: CALCULUS II

Test Three – April 3, 2003 – 75 Minutes

Part II

No books, notes or calculators. This part is long-answer and hand-marked. Show your work.

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1. An error bound for the approximation to $\int_a^b f(x)\,dx$ given by Simpson's Rule with n subintervals is the following:

$$|E_S| \leq \frac{K(b-a)^5}{180n^4} \text{ where } |f^{(4)}(x)| \leq K \text{ on } [a,b].$$

According to this error bound, how many subintervals do we need to obtain an approximation to $\int_1^4 \frac{1}{x}\,dx$ accurate to within 0.00004? [3 points]

2. Use the Comparison Test to determine whether the improper integral $\int_1^\infty e^{-x^2}\,dx$ is convergent or divergent. Justify your answer. [3 points]

3. Find the surface area of the ellipsoid obtained by revolving the ellipse $\frac{x^2}{4} + y^2 = 1$ about the x-axis. Sketch the ellipsoid. [6 points]

4. Find the arc length function for the curve $y = (x-1)^{3/2}$ with the starting point $(1, 0)$. [4 points]

5. Show that every function of the form $y = Ae^x + Bxe^x$, where A and B are arbitrary constants, is a solution of the differential equation $y'' - 2y' + y = 0$. [2 points]

6. Consider the differential equation $\frac{dy}{dx} = 4x^3y$.

   (a) Find the general solution of the equation. [3 points]

   (b) Find the particular solution that satisfies $y(0) = -1$. [1 point]